

In flight calibration plan for the instrument of the MICROSCOPE space mission

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retour sur innovation

Origin and objectives

Expressed in 1911 by Einstein, the Equivalence Principle is:

- at the basis of the General Relativity
- only justified by experimental observations: specifically, universality of free fall → all bodies, independently of their mass or intrinsic composition, acquire the same acceleration in the same uniform gravity field

Recent Results:

- in laboratory (Adelberger, Phys. Rev. 1990; Su Y, Phys. Rev. 1994; S. Baessler et al, Phys. Rev. Let. 1999) 10⁻¹² to 10⁻¹³
- by Moon Earth Sun system observation (Williams et al., Int.J.Mod.Phys.D18:1129-1175,2009)→ 10⁻¹²

Difficulties to merge:

- General Relativity & Quantum Theory
- Gravitational Interaction & Standard Model
- \rightarrow equivalence principle violation below 10⁻¹⁴
 - (T. Damour et al. Nucl. Phys. Rev D, 046007, 2002)

The PE tests provide a more simple and direct check than the other tests of the General Relativity

The Equivalence Principle



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The principle of the MICROSCOPE space mission



MICROSCOPE Mission Main Parameters





- 2 Differential accelerometers (SU-EP and SU-Ref)
- 2 Front End Electronic Unit
- 2 Interface and Control Unit Mechanical Ensemble + software
- The harnesses

Sensor Unit



Control Electronics



Principle of an electrostatic accelerometer

- Proof mass
 - Physically 'free', except a gold wire of 5µm
 - Surounded by electrods (cylindrical configuration)
 - Electrostatic potential controled by the gold wire
 - Vd sine to detect the position
 - Vp constant to apply a certain force
 - Kept centered and motionless by control loop
- The electrods enable
 - The measurement of the position of the mass through variation of capacitance
 - The control of the electrostatic fields which control the mass
 - The control of its 6 degrees of freedom with electrostatic forces



Measurement principle

Ideal measurement of of a proof mass (k):

$$\vec{\Gamma}_{App,k} = \frac{M_{gsat}}{M_{Isat}} \vec{g}(O_{sat}) - \frac{m_{gk}}{m_{Ik}} \vec{g}_k + \frac{\vec{F}_{NGsat}}{M_{Isat}} - \frac{\vec{F}_{Pak}}{m_{Ik}} + R_{In,Cor}(\overrightarrow{O_{sat}O_k})$$



Half difference of acceleration of the two masses:

$$\vec{\Gamma}_{d} \approx \frac{1}{2} \Big(\delta \vec{g}(O_{sat}) + \big(\mathbf{T} - \mathbf{I}\big) \overline{O_{1}O_{2}} \Big) \qquad \qquad \delta = \frac{m_{2g}}{m_{2I}} - \frac{m_{1g}}{m_{1I}}$$

Measurement limitation

• Real Measured acceleration of a proof mass (*k*):

$$\vec{\Gamma}_{meas,k} = \vec{B}_{o,k} + \mathbf{M}_{k}\vec{\Gamma}_{k} + \sum_{u=x,y,z} \left(\hat{u}^{T} \cdot \left(\mathbf{K}_{2,k} \cdot \vec{\Gamma}_{k} \cdot \vec{\Gamma}_{k}^{T}\right) \cdot \hat{u}\right) \cdot \hat{u} + \vec{\Gamma}_{n,k}$$

$$\vec{\Gamma}_{mes,k} = \vec{b}_{0k} + \left(\left[1 + \frac{dK_{1k}}{scale}\right] + \left[\frac{\eta_{k}}{coupl}\right]\right) \cdot \left[\frac{\theta_{k}}{align}\right] \cdot \left(\vec{\Gamma}_{app,k/sat} + \frac{\vec{F}_{ext/sat}}{M_{Isat}} + \frac{\vec{F}_{th/sat}}{M_{Isat}}\right)$$

$$+ \left(\left[1 + \frac{dK_{1k}}{scale}\right] + \left[\frac{\eta_{k}}{coupl}\right]\right) \cdot \left(-\frac{\vec{F}_{pak/inst,k}}{m_{Ik}} - \frac{\vec{F}_{el,park/inst,k}}{m_{Ik}}\right)$$

$$+ \frac{K_{2k}}{quad} \Gamma_{App,k/sat}^{2}$$

Expression of the differential measurement

- The test is performed at $f_{ep} \rightarrow$ the bias can be neglected
- The common acceleration corresponds to the residual of the drag free system and its command
- The differential acceleration is composed of the violation signal + the Earth gravity gradient and inertia tensor terms due to the off-centring of the masses

$$\Gamma_{mes,dx} = \frac{1}{2} K_{1cx} \cdot \delta \cdot g_{x/sat} + \frac{1}{2} \begin{bmatrix} K_{1cx} \\ \eta_{cz} + \theta_{cz} \\ \eta_{cy} - \theta_{cy} \end{bmatrix}^{t} \cdot \begin{bmatrix} \Delta_x \\ \Delta_y \\ \Delta_z \end{bmatrix}$$
 Inertia tensor
Inertia tensor
Drag free residual
$$+ \begin{bmatrix} K_{1dx} \\ \eta_{dz} + \theta_{dz} \\ \eta_{dy} - \theta_{dy} \end{bmatrix}^{t} \cdot (\vec{\Gamma}_{res_{df}} + C_x) + 2 \cdot K_{2cxx} \cdot (\Gamma_{app,dx} + b_{1dx}) \cdot (\Gamma_{res_{df},x} + C_x - b_{0cx})$$

$$+ K_{2dxx} \cdot \left((\Gamma_{res_{df},x} + C_x - b_{0cx})^2 + (\Gamma_{app,dx} + b_{1dx})^2 \right)$$
 Drag free
command

Objective of accuracy

- Global objective: 10^{-15} g \rightarrow 8×10⁻¹⁵ ms⁻²
- Objective for $\Gamma_{mes,dx}$: 4×10⁻¹⁵ ms⁻²
- Source of errors: mechanical defects, gravity gradient, thermal and magnetic effects
 → 40 groups of contributors
- Specification for each group: 10⁻¹⁶ ms⁻²
- 3 groups are explicit in the measurement equation

Budget before calibration

	Signal element	Parameter concerned	Contribution before calibration (m·s ⁻²)	
Defects between the instrument and the satellite	$K_{1cx} \cdot T_{xx} \cdot \Delta_x$	$K_{1cx} \cdot \Delta_x < 20.2 \mu\mathrm{m}$	8.4×10 ⁻¹⁴	
	$K_{1cx} \cdot T_{xz} \cdot \Delta_z$	$K_{1cx} \cdot \Delta_z < 20.2 \mu\mathrm{m}$	8.6×10 ⁻¹⁴	
	$K_{1cx} \cdot T_{xy} \cdot \Delta_y$	$K_{1cx} \cdot \Delta y < 20.2 \mu\mathrm{m}$	6×10 ⁻¹⁶	
	$(\eta_{cz} + \theta_{cz}) \cdot T_{yy} \cdot \Delta_y$	$\eta_{cz} + \theta_{cz} < 2.6 \times 10^{-3} \text{ rad}$	8.6×10 ⁻¹⁶	
		$\Delta_y < 20 \mu m$		
	$(\eta_{cy} - \theta_{cy}) \cdot T_{zz} \cdot \Delta_z$	$\eta_{cy} - \theta_{cy} < 2.6 \times 10^{-3} \text{ rad}$	6.4×10 ⁻¹⁶	
C		$\Delta_z < 20 \mu m$		
Defects between the two sensors	$2 \cdot K_{1dx} \cdot \Gamma_{res_{df},x}$	$K_{1dx} < 10^{-2}$	2×10 ⁻¹⁴	
	$2 \cdot (\eta_{dz} + \theta_{dz}) \cdot \Gamma_{res_{df}, y}$	$\eta_{dz} + \theta_{dz} < 1.6 \times 10^{-3} \text{ rad}$	3.0×10 ⁻¹⁵	A
	$2 \cdot (\eta_{dy} - \theta_{dy}) \cdot \Gamma_{res_{df},z}$	$\eta_{dy} - \theta_{dy} < 1.6 \times 10^{-3}$ rad	3.0×10 ⁻¹⁵	posteriori
Quadratic non {	$4 \cdot K_{2,cxx} \cdot \Gamma_{app,dx} \cdot \Gamma_{res_{df},x}$	$K_{2cxx} < 20000 \text{ s}^{2/\text{m}}$	8.0×10 ⁻¹⁶	is required
	$2 \cdot K_{2,dxx} \cdot \left(\Gamma_{res_{df},x}^2 + \Gamma_{app,dx}^2 \right)$	K_{2dxx} < 20000 s ² /m	8.0×10 ⁻¹⁶	
	Total = \sum		2×10 ⁻¹³	

Overview of the calibration

Objective of accuracy of the correction:

$$\Gamma_{mes,d} - \frac{1}{2}\delta g - \Sigma \hat{E}_i = \Sigma \delta E_i < 3 \times 10^{-16} \, ms^{-2}$$

- Each group is specified to be $< 10^{-16} ms^{-2}$
- Calibration of some parameters to correct the measurement
 → Create a signal that amplifies the effect of the parameter to be calibrated in Γ_{mes,d}
- Reduction of the noise limiting the calibration through integration over several orbits
- Improvement of the calibration performance by reassessing the estimation of a parameter using the estimated value of others

Defects between the instrument and the satellite

$$\Gamma_{mes,dx} = \frac{1}{2} K_{1cx} \cdot \delta \cdot g_{x/sat} + \frac{1}{2} \begin{bmatrix} K_{1cx} \\ \eta_{cz} + \theta_{cz} \\ \eta_{cy} - \theta_{cy} \end{bmatrix}^{t} \cdot [T - In] \cdot \begin{bmatrix} \Delta_{x} \\ \Delta_{y} \\ \Delta_{z} \end{bmatrix} + \begin{bmatrix} K_{1dx} \\ \eta_{dz} + \theta_{dz} \\ \eta_{dy} - \theta_{dy} \end{bmatrix}^{t} \cdot (\vec{\Gamma}_{res_{df}} + \vec{C}) + 2 \cdot K_{2cxx} \cdot (\Gamma_{app,dx} + b_{1dx}) \cdot (\Gamma_{res_{df},x} + C_{x} - b_{0cx}) + K_{2dxx} \cdot ((\Gamma_{res_{df},x} + C_{x} - b_{0cx})^{2} + (\Gamma_{app,dx} + b_{1dx})^{2})$$

Signal element $K_{1cx} \cdot \Delta_x \cdot T_{xx}$ $K_{1cx} \cdot \Delta_z \cdot T_{xz}$ $K_{1cx} \cdot \Delta_y \cdot T_{xy}$ $(\eta_{cz} + \theta_{cz}) \cdot T_{yy} \cdot \Delta_y$ $(\eta_{cy} - \theta_{cy}) \cdot T_{zz} \cdot \Delta_z$

Use the important value of T_{xx} and T_{xz} at $2f_{orb}$. For $K_{1cx} \cdot \Delta_x$: $\Gamma_{calib_1} = 2 \cdot \Gamma_{mes,dx/cos}(2f_{orb}) = T_{xx}(2f_{orb}) \cdot K_{1cx} \cdot \Delta_x$

 $\begin{array}{l} \mathsf{T}_{xy} \text{ is too weak} \rightarrow \text{ oscillate the} \\ \text{satellite around } \mathsf{Y}_{\mathsf{sat}} \text{:} \quad \Gamma_{calib_1} \ = \ 2 \cdot \Gamma_{mes,dx}(f_{cal}) = \alpha_0 \cdot \left[\begin{array}{c} \uparrow \\ T_{yy}(DC) - T_{xx}(DC) - \omega_{cal}^2 \end{array} \right] \cdot K_{1cx} \Delta_y \end{array}$

 $= \left(\eta_{cy} - \theta_{cy}\right) \cdot \frac{\overbrace{\Delta_{TM} \cdot (1 + g_{det_y})}^{\wedge}}{2} \cdot \left(-2\alpha_0 \omega_{TM} \omega_{cal} + \alpha_0 \left[\stackrel{\circ}{T_{zz}} (DC) - \stackrel{\circ}{T_{yy}} (DC) + \omega^2_{cal} \right] \right)$

Oscillate the satellite around an axis and oscillate the mass along an other axis \rightarrow Coriolis effect. For $(\eta_{cy} - \theta_{cy})$: $\Gamma_{calib_1} = 2 \cdot \Gamma_{mes,dx}(f_{cor})$

 Δx , y, z cannot be reduced T is very well known

Parameter to be calibrated Signal amplifier

Defects between the two sensors

$$\Gamma_{mes,dx} = \frac{1}{2} K_{1cx} \cdot \delta \cdot g_{x/sat} + \frac{1}{2} \begin{bmatrix} K_{1cx} \\ \eta_{cz} + \theta_{cz} \\ \eta_{cy} - \theta_{cy} \end{bmatrix}^{t} \cdot [T - In] \cdot \begin{bmatrix} \Delta_{x} \\ \Delta_{y} \\ \Delta_{z} \end{bmatrix} + \begin{bmatrix} K_{1dx} \\ \eta_{dz} + \theta_{dz} \\ \eta_{dy} - \theta_{dy} \end{bmatrix}^{t} \cdot [\tilde{r}_{res,g} + \tilde{C}] + 2 \cdot K_{2cxx} \cdot (\Gamma_{app,dx} + b_{1dx}) \cdot (\Gamma_{res,g,x} + C_{x} - b_{0cx} + K_{2dxx} \cdot (\Gamma_{res,g,x} + C_{x} - b_{0cx})^{t} + (\Gamma_{app,dx} + b_{1dx})^{2}) + K_{2dxx} \cdot (\Gamma_{res,g,x} + C_{x} - b_{0cx})^{t} + (\Gamma_{app,dx} + b_{1dx})^{2}) + (\Gamma_{app,dx} + b_{1dx})^{2} + (\Gamma_{app,dx} + b_{1dx})^{2}) + (\Gamma_{app,dx} + b_{1dx})^{2} + (\Gamma_{app,dx} + h_{1dx})^{2} + (\Gamma_{app,dx} + h_{1dx$$

Quadratic non linearities

$$\Gamma_{mes,1x} = \begin{bmatrix} K_{11x} \\ \eta_{1z} + \theta_{1z} \\ \eta_{1y} - \theta_{1y} \end{bmatrix}^t \cdot \Gamma_{app,1/sat} + K_{21xx} \cdot \vec{\Gamma}_{App,1x/sat}^2$$



Oscillate the satellite along X_{inst}. The measured acceleration is controlled to follow a sine $\Gamma_{calib1} = \Gamma_{mes,dx}(2f_{cal}) = \frac{1}{2} \frac{K_{2dxx}}{K_{1cx}^2} \left(\Gamma_{mes,cx}(f_{cal})^2\right)^2$ Calibrate K₂₁ and K₂₂. For K₂₁ (resp. K₂₂), drag compensation locked on the sensor 2 (resp. 1) and oscillation of the proof mass 1 (resp. 2) along X_{inst} $\Gamma_{calib1} = \Gamma_{mes,1x}(2f'_{TM}) = \frac{1}{2} \frac{K_{21xx}}{K_{12x}^2} \Gamma^2_{mes,1x}(f'_{TM})$ $\Gamma_{calib1} = \Gamma_{mes,2x}(2f'_{TM}) = \frac{1}{2} \frac{K_{22xx}}{K_{12x}^2} \Gamma^2_{mes,2x}(f'_{TM})$

Exemple of a calibration process: $K_{1cx}\Delta x$

Global equation

$$2 \cdot \Gamma_{mes,dx/\cos}(2f_{orb}) \approx \overline{T_{xx}(2f_{orb})} \cdot \overline{K_{1cx}} \cdot \Delta_x + (\eta_{cy} - \theta_{cy}) \cdot T_{zz}(2f_{orb}) \cdot \Delta_z + 2 \cdot K_{1dx} \cdot \Gamma_{res_{df},x}(2f_{orb}) + 2 \cdot (\eta_{dz} + \theta_{dz}) \cdot \Gamma_{res_{df},y}(2f_{orb}) + 2 \cdot (\eta_{dy} - \theta_{dy}) \cdot \Gamma_{res_{df},z}(2f_{orb}) + 4 \cdot K_{2cxx} \cdot \left\{ \Gamma_{app,dx} + b_{1dx} \right\} \cdot \left(\Gamma_{res_{df},x} + C_x - b_{0cx} \right) \right\} (2f_{orb}) + 2 \cdot \Gamma_{n,dx}$$

• The calibration equation is:
$$\Gamma_{calib1} = 2 \cdot \Gamma_{mes,dx/cos}(2f_{orb}) = T_{xx}(2f_{orb}) \cdot K_{1cx} \cdot \Delta_x$$

- Some contributors lead to a systematic error and a stochastic error. The stochastic limitation is accounted for as: $3\frac{DSP_{noise}}{\sqrt{T_{cal}}}$
- The 2nd iteration calibration equation is:

$$\Gamma_{calib2} = 2 \cdot \Gamma_{mes,dx/\cos}(2f_{orb}) - (\overline{\eta_{cy} - \theta_{cy}}) \cdot T_{zz}(2f_{orb}) \cdot \Delta_{z} - 2 \cdot K_{1dx} \cdot \Gamma_{mes,cx}(2f_{orb})$$
$$-2 \cdot (\overline{\eta_{dz} + \theta_{dz}}) \cdot \Gamma_{mes,cy}(2f_{orb}) - 2 \cdot (\overline{\eta_{dy} - \theta_{dy}}) \cdot \Gamma_{mes,cz}(2f_{orb})$$
$$= T_{xx}(2f_{orb}) \cdot K_{1cx} \cdot \Delta_{x}$$

Evaluated calibration budget

T _{cal} = 10 orbits	Parameter to be calibrated	Perfo. After 1 round of calibration	Perfo. After 2 round of calibration	Specification
Calibrated at 1st iteration	$K_{1cx} \cdot \Delta_x$	0.15 µm	0.08 µm	0.1 µm
	$K_{1cx} \cdot \Delta_z$	0.16 µm	0.08 µm	0.1 µm
	$K_{1cx} \cdot \Delta_y$	1.3 µm	1.3 µm	2 µm
	$(\eta_{cz} + \theta_{cz})$	1.7×10 ⁻³ rad	1.1×10 ⁻³ rad	1.0×10 ⁻³ rad
Unsufficiently calibrated To be corrected with non linear term: evaluate (K1+2K2b1)d the actual sensitivity about the b1	$\left(\eta_{cy}-\theta_{cy}\right)$	1.7×10 ⁻³ rad	1.0×10 ⁻³ rad	1.0×10 ⁻³ rad
	K_{1dx}/K_{1cx}	2.3×10 ⁻³	2.3×10 ⁻³	1.5·10 ⁻⁴
	$\frac{\eta_{dz} + \theta_{dz}}{K_{1cy}}$	5.9×10 ⁻⁵ rad	1.7×10 ⁻⁵ rad	5.10 ⁻⁵ rad
	$\frac{\eta_{dy} - \theta_{dy}}{K_{1cz}}$	5.9×10 ⁻⁵ rad	1.7×10 ⁻⁵ rad	5.10 ⁻⁵ rad
	K_{2dxx}/K_{1cx}^2	1206.6 s²/m	188.7 s²/m	250 s²/m
	K_{2cxx}/K_{1cx}^2	271.7 s²/m	271.7 s²/m	250 s²/m
acceleration level				

Conclusion (1/2)

- Payload:
 - Integration of the qualification model of the accelerometer realized
 - 06-09/2010: Free fall tests in Bremen
 - 10-11/2010: Qualification tests with the QM: vibrations, thermal vacuum, shock
 - 01-06/2011: flight model, tests
- Satellite:
 - Study of the cold gas propulsion
 - 2011: Preliminary definition of the flight model
 - 2012: Definition of the FM
 - 2013: Integration of the payload
 - 2014: launch

Conclusion (2/2)

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- Calibration process definition
 - The budget of the measurement equation before calibration does not comply with the objective of the EP test accuracy
 - Several in flight calibrations are necessary during the space experiment
 - Parameters to be calibrated have been identified and appropriate methods of calibration have been proposed
 - The calibration accuracy has been analytically evaluated.
 All parameters are estimated with sufficient accuracy → differential scale factor to be more investigated taking into account non linearity
 - Next step: simulation of the calibration with satellite attitude and drag-free system

Sensor Unit (2/2)

