Review of analytical studies of relative motions in flight formation

by

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under direction of

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MISSION	DESCRIPTIF	MISSION STATUS	ORBIT
GRACE	Two satellites to study Earth gravity field. Distance measurements.	Launched in 2002	a = 6683 km, e = 0.0022 , i = 89°
CALIPSO /CLOUDSAT	Integrated in the A-TRAIN for Earth observation. The same ground track.	2006	heliosynchronius, h = 705 km, i = 98.2 °
DARWIN	Four satellites for the exoplanets detection. Interferometry.	phase A, launching 2015 ?	at L2
LISA	Three satellites for gravity waves detection. Interferometry.	launching 2014 ?	heliocentric, 20° after the Earth, relative distances 5 millions of km
TPF	Equivalent to DARWIN.	launching 2020	at L2
ASPICS	Two satellites to study sun's corona.	proposition at CNES	?
SIMBOL-X	High focal distance telescope to observe high energy radiation. 2 satellites.	proposition at CNES	h = 81500 km
PEGASE	Spatial interfermeter to detect exoplanets type Jupiter.	proposition at CNES	at L2
MAX	High energy spectrometer. Two satellites to increase focal distance.	proposition at CNES	at L2
PRISMA	Technologic mission.	launching 2008	heliosynchronius, h = 700 km
ROMULUS	Spatial radar to observe mobile targets.	?	LEO
TanDEM-X/ TerraSAR-X	High precision radar interferometry for terrestrial altimetry.	launching 2009	heliosynchronius, h = 515 km

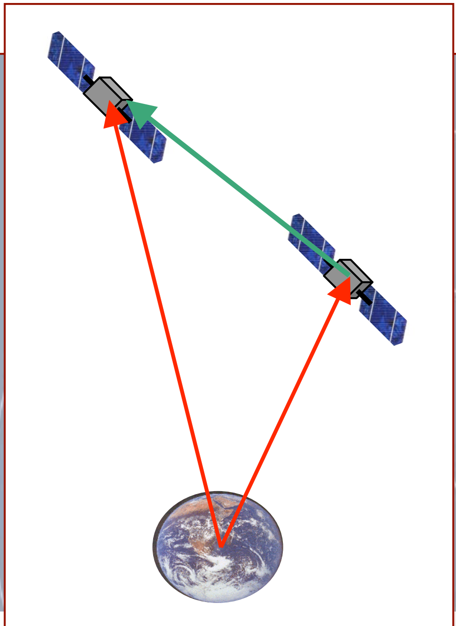
1.1 Flight Formations

- **What is a flight formation?**
  - **Little intersatellite distances**
  - **Major role of relative motion and velocity**
- **Which is the difference with a constellation?**
  - **Constellation : ‘two or more spacecraft in similar orbits with no active control by either to maintain a relative position’**
  - **Flight Formation : ‘Formation flight involves the use of an active control scheme to maintain the relative positions of the spacecraft’**
- **Why Flight Formations are interesting?**
  - **Bigger ‘virtual satellites’**
  - **Reduction of launching risks**
  - **Reduction of mass and mission cost**

Flight Formations

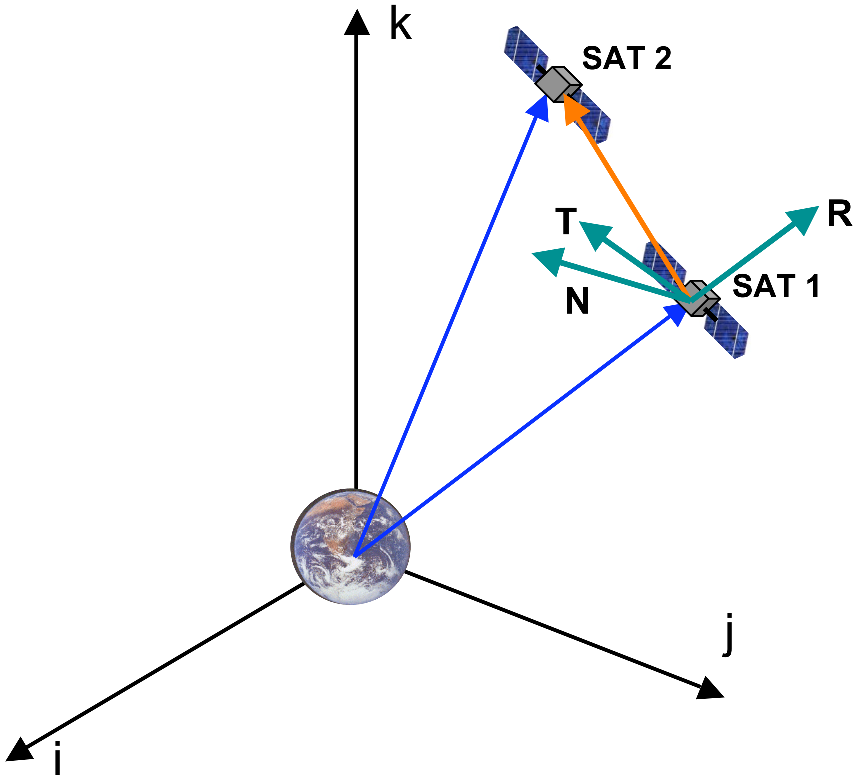
- **At Lagrange points** — Inertial Dynamics (DI)
  - Small intersatellite distance
    - DARWIN
    - TPF
    - PEGASE
    - MAX
  - Big intersatellite distance
    - LISA
- **Around the Earth** — Central Body Dynamics (DCC)
  - GRACE
  - CALIPSO / CLOUDSAT
  - ASPICS
  - TechSat 21
  - SIMBOL - X

Boutonnet (2003)



- **RELATIVE MOVEMENTS**
- **HILL'S EQUATIONS**
  - **Also known as Clohessy-Wiltshire equations**
  - **Historically used for rendezvous problem**
  - **Lawden's equations: eccentric orbits**
- **ORBITAL ELEMENTS**

THE ACTORS OF RELATIVE MOVEMENT



$$EO : a, e, i, \Omega, \omega, M$$

- **SAT 1:** $(\vec{r}_1|_{IJK}, \vec{v}_1|_{IJK}) \equiv \vec{x}_1|_{IJK}$
  EO_1
- **SAT 2:** $(\vec{r}_2|_{IJK}, \vec{v}_2|_{IJK}) \equiv \vec{x}_2|_{IJK}$
  EO_2
- **DIFF:** $\Delta\vec{x}|_{IJK} = \vec{x}_2|_{IJK} - \vec{x}_1|_{IJK}$
  $\Delta EO = EO_2 - EO_1$
- **RTN:** $\delta_R, \delta_T, \delta_N$ $\quad \Delta\vec{r}|_{RTN} = \vec{\delta}$
  $\dot{\delta_R}, \dot{\delta_T}, \dot{\delta_N}$ $\quad \Delta\vec{v}|_{RTN} \neq \dot{\vec{\delta}}$

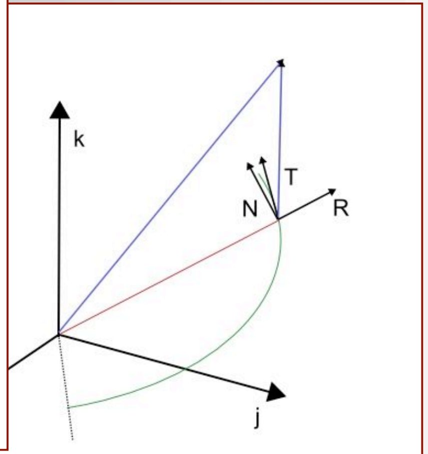
▫ HILL'S EQUATIONS

- use of a circular reference frame
- integration of resulting differential equations

$$\begin{aligned}
\ddot{\delta}_R &= 3n_0^2\delta_R + 2n_0\dot{\delta}_T + F_R \\
\ddot{\delta}_T &= -2n_0\dot{\delta}_R + F_T \\
\ddot{\delta}_N &= -n_0^2\delta_N + F_N
\end{aligned}$$

$$\begin{aligned}
\delta_R(t) &= \frac{\dot{\delta}_R(0)}{n_0}\sin n_0 t - \left(2\frac{\dot{\delta}_T(0)}{n_0} + 3\delta_R(0)\right)\cos n_0 t \\
&+ \left(2\frac{\dot{\delta}_T(0)}{n_0} + 4\delta_R(0)\right) \\
\delta_T(t) &= 2\frac{\dot{\delta}_R(0)}{n_0}\cos n_0 t + \left(4\frac{\dot{\delta}_T(0)}{n_0} + 6\delta_R(0)\right)\sin n_0 t \\
&+ \left(-2\frac{\dot{\delta}_R(0)}{n_0} + \delta_T(0)\right) - \left(3\dot{\delta}_T(0) + 6n_0\delta_R(0)\right)t \\
\delta_N(t) &= \delta_N(0)\cos n_0 t + \frac{\dot{\delta}_N(0)}{n_0}\sin n_0 t
\end{aligned}$$

(Hill, 1967)



POSITION AND VELOCITY WITH RESPECT TO HILL REFERENCE FRAME

DEVELOPMENTS ABOUT HILL EQUATIONS

- **J2 effect**
  - *Schweighart, S. & Sedwick, R.:*
    A perturbative analysis of geopotential disturbances for satellite cluster formation flying
    IEEE Aerospace Conference Proceedings, Big Sky, MT, 10-17 Mar. 2001
  - *Xu, C.; Tsoi, R. & Sneeuw, N.:*
    Analysis of J2 perturbed relative orbits for satellite formation flying
    Gravity, Geoid and Space Missions -- GGSM2004
    Jekeli, C., Bastos, L. & Fernandes, J.*(Ed.)*
    IAG proceedings, Springer 2005 , Volume 129 , pp. 36-41

- **High-order solutions**
  - *Gomez, G. & Marcote, M.*
    High-order analytical solutions of hill's equations
    Celestial Mechanics and Dynamical Astronomy 2006, vol. 94, pp 197-211

LAWDEN'S EQUATIONS

- **Equations for eccentric orbits :** *Lawden, D. F.:*
  Optimal Trajectories for Space Navigation
  Butterworths, London, England, 1963

$$\begin{aligned}
\delta_R(t) &= A\cos f + Be\sin f + CI_2 \\
\delta_T(t) &= -A\sin f + B(1+e\cos f) + \frac{D - A\sin f}{1+e\cos f} + CI_2 \\
\delta_N(t) &= \frac{1}{1+e\cos f}(E\cos f + F\sin f)
\end{aligned}$$

$$\begin{aligned}
I_2 &= \frac{\cot f}{e(1+e\cos f)} + \frac{1+e\cos f}{e\sin f} I_1 \\
I_1 &= \sin f \int \frac{df}{\sin^2 f (1+e\cos f)^2}
\end{aligned}$$

DEVELOPMENTS ABOUT LAWDEN EQUATIONS

Use of homogeneous solution for control optimisation:

- *Carter, T. E.:*
  New Form for the optimal rendezvous equations near a keplerian orbit
  J. Guidance, vol. 13, No.1, Jan-Feb. 1990
- *Carter, T. & Humi, M.:*
  Fuel-optimal rendezvous near a point in general keplerian orbit
  J. Guidance, vol. 10, No. 6, Nov.-Dec., 1987
- *Inalhan, G.; Tillerson, M. & How, J.P.:*
  Relative Dynamics and Control of Spacecraft Formations in Eccentric Orbits
  J. of Guidance, Control and Dynamics, Vol. 25, No.1, Jan;-Feb; 2002
- *Tillerson, M. & How, J. P.:*
  Formation Flying Control in Eccentric Orbits
  Proceedings of the AIAA Guidance, Navigation, and Control Conference, Montreal, August 2001.

WORKING WITH ORBITAL ELEMENTS

- **Many theories are available for temporal evolution of orbital elements, so, let us use them !**
- **If distances are small enough, they can be linearised for the orbital element differences**
- **If distances are not small enough, another kind of non-linear transformation will be necessary**

$$EO = f(EO_0, t)$$

$$\Delta EO = \frac{df(EO_0, t)}{dEO_0} \Delta EO_0$$

WORKING WITH ORBITAL ELEMENTS

KEPLERIAN MOVEMENT:

$$
\begin{aligned}
EO &= EO_0 & \Delta EO &= \Delta EO_0 \\
M &= M_0 + n_0\,(t - t_0) & \Delta M &= \Delta M_0 + \Delta n_0\,(t - t_0)
\end{aligned}
$$

SECULAR J2 EFFECT IN ASCENDING NODE:

$$
\Omega = \Omega_0 - \frac{3}{2} J_2 R_T^2 \sqrt{\mu} \frac{\cos i_0}{a_0^{7/2}} \frac{1}{(1 - e_0^2)^2} (t - t_0)
$$

$$
\Delta\Omega = \Delta\Omega_0 + \left[\frac{d}{da}\left(\frac{d\Omega}{dt}\right) \Delta a_0 + \frac{d}{de}\left(\frac{d\Omega}{dt}\right) \Delta e_0 + \frac{d}{di}\left(\frac{d\Omega}{dt}\right) \Delta i_0 \right] (t - t_0)
$$

$$
\Delta\Omega = \Delta\Omega_0 - \frac{3}{2} J_2 R_T^2 \sqrt{\mu} \frac{\cos i_0}{a_0^{7/2}} \frac{1}{(1 - e_0^2)^2} (t - t_0) \left(-\frac{7}{2a_0} \Delta a_0 + \frac{4e_0}{1 - e_0^2} \Delta e_0 - \tan i_0 \Delta i_0 \right)
$$

- **Transformations between position and velocity differences and orbital element differences**
  - **As we are interested in position and velocity, it will be necessary to transform the differences of orbital element in differences on position and velocity.**

$$\Delta\overrightarrow{x} = f(EO, \Delta EO)$$

$$\Delta EO = f^{-1}(EO, \Delta\overrightarrow{x})$$

$$\Delta\overrightarrow{x} = M(EO)\Delta EO$$

$$\Delta EO = M^{-1}(EO)\Delta\overrightarrow{x}$$

- *Casotto, S.:*
  Position and velocity perturbations in the orbital frame in terms of classical element perturbations
  Celestial Mechanics and Dynamical Astronomy 55: 209,221,1993

- *Sabol, C.; McLaughlin, C. A. & Kim Luu, K.:*
  Meet the cluster orbits with perturbations of keplerian elements (COWPOKE) equations
  AAS/AIAA Space Flight Mechanics Meeting, Ponce, Puerto Rico, 9-13 February 2003, Paper AAS 03-138

Previous work

$$EO = f(EO_0, t)$$

$$\Delta EO = \frac{df(EO_0, t)}{dEO_0} \Delta EO_0$$

$$\Delta \overrightarrow{x} = M(EO) \Delta EO$$

→

$$\Delta \overrightarrow{x} = f(EO, \Delta EO_0, t)$$

- *Schaub, H.:*
  Hybrid Cartesian and Orbit element feedback law for formation flying spacecraft
  J. of Guidance, Navigation and Control, Vol. 25, N.2, March-April, 2002 pp 387-393

- *Alfriend, K. T.; Schaub, H. & Gim, D.*
  Gravitational perturbations, nonlinearity and circular orbit assumption effects on formation flying strategies
  AAS Guidance and Control Conference, Breckenridge, CO, Feb. 2-6, 2000

- *Schaub, H.:*
  Spacecraft relative orbit geometry description through orbit element differences
  14TH U.S. National Congress of Theoretical and Applied Mechanics, Blacksburg, VA, June 23-28, 2002

Our method:

$$EO = f(EO_0, t)$$

$$\Delta EO = \frac{df(EO_0, t)}{dEO_0} \Delta EO_0$$

$$\Delta \overrightarrow{x} = M(EO) \Delta EO$$

$$\Delta EO = M^{-1}(EO) \Delta \overrightarrow{x}$$

→

$$\Delta \overrightarrow{x} = f(EO, \Delta \overrightarrow{x}_0, t)$$

- **First step:** differences of orbital elements as fonction of position and velocity
- **Second step:** application to the keplerian movement
- **Third step:** introduction of perturbations

First step

- **Direct inversion of the matrix M is not possible**
- **Direct derivation of relations between orbital elements and position/velocity is not possible because there are no explicit expressions**
- **It is convenient to use the properties of Poisson brackets**

$$\frac{dEO_i}{dq_j} = \sum_{K=1}^{6} \frac{dp_j}{dEO_K}\{EO_i, EO_K\}$$

$$\frac{dEO_i}{dp_j} = -\sum_{K=1}^{6} \frac{dq_j}{dEO_K}\{EO_i, EO_K\}$$

↓

$$\Delta EO = M^{-1}(EO)\Delta\overrightarrow{x}$$

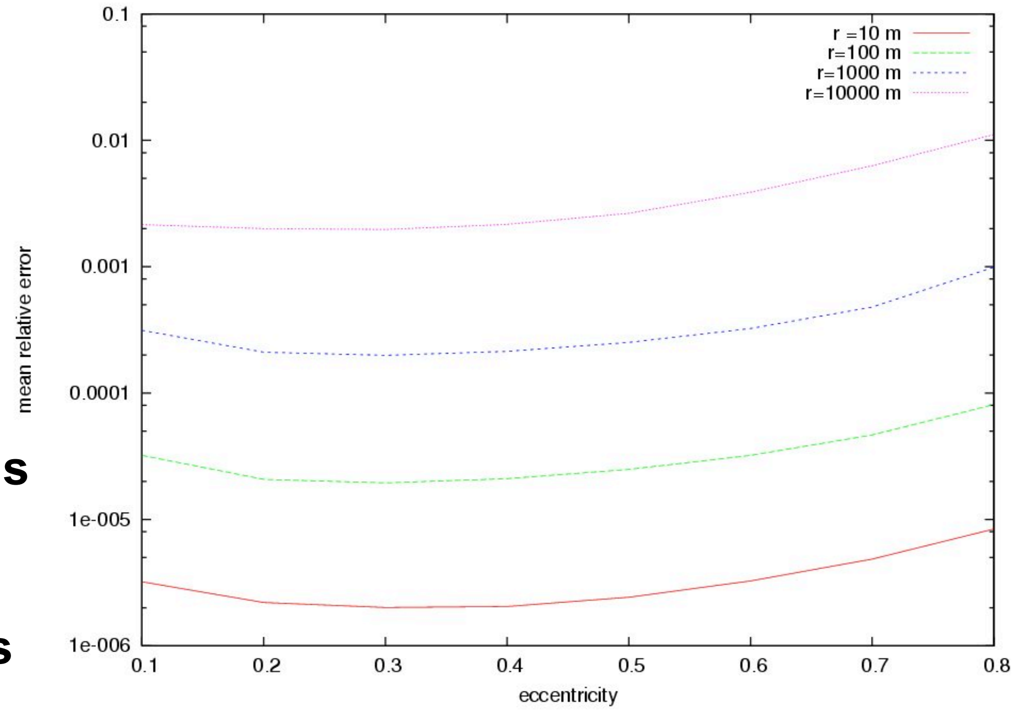
Second step: Keplerian movement

$$\Delta R(t) = A_R\left(\frac{r}{a}\right) + B_R \sin f + C_R \cos f + D_R t \sin f$$

$$\Delta T(t) = A_T\left(\frac{r}{a}\right) + B_T \sin f + C_T\left(\frac{a}{r}\right) + D_T\left(\frac{r}{a}\right)\sin f + E_T\left(\frac{a}{r}\right)t$$

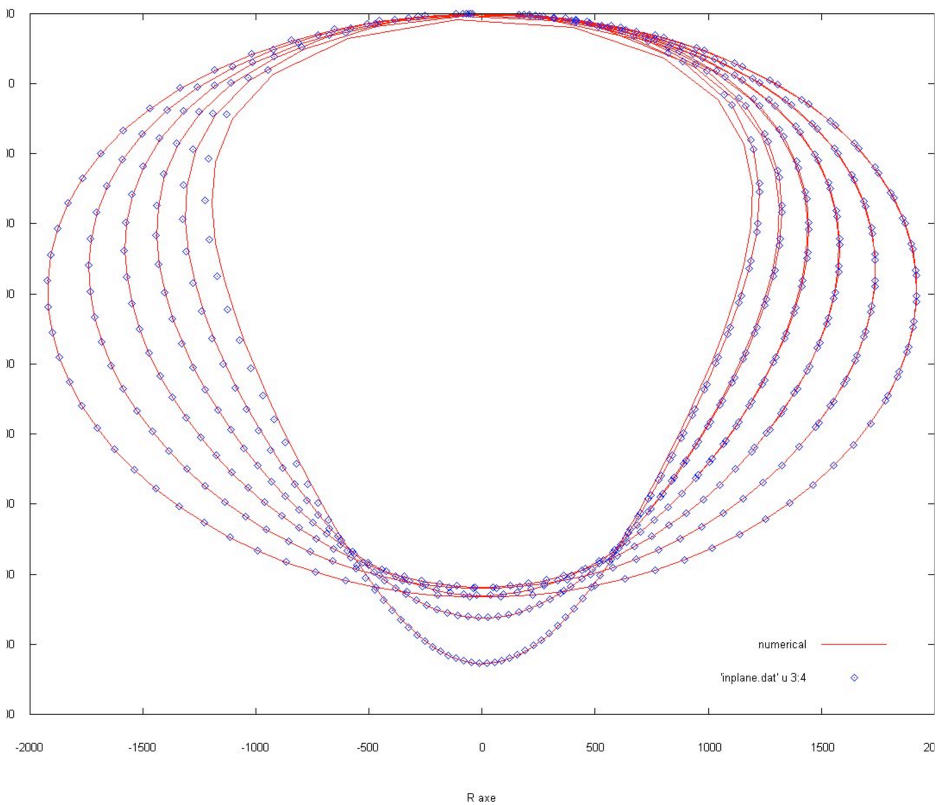
$$\Delta N(t) = \frac{r}{\beta^2 a}\left(\cos(f-f_0) + e\cos f\right)\Delta N_0 + \frac{1}{n\beta}\frac{r}{a}\frac{r_0}{a}(1-e\cos f_0)\sin(f-f_0)\Delta V_{N0}$$

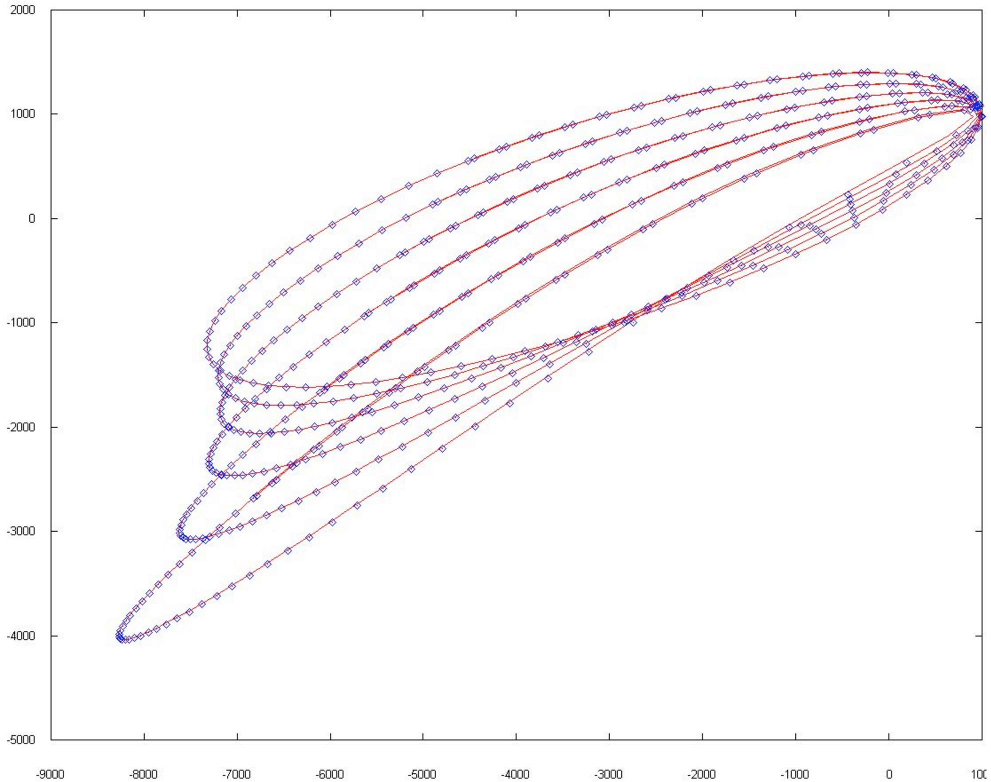
$$A_R, B_R, C_R, D_R, A_T, B_T, C_T, D_T, E_T : f(EO_0, \Delta\vec{x}_0)$$



- comparable to Lawden equations
- valid for all eccentricities
- error related to initial conditions

▫ Numerical validation of analytical expressions





- **inplane movement (R,T)**
- **out of plane movement (T,N)**

Further work:

- Third step: introduction of perturbations, gravity field
- Identification of secular drifts for each kind of perturbation
- Optimisation of configuration for different missions

Conclusions:

- The use of orbital elements is more adapted than relative position and velocity
- To describe the movement in the RTN reference frame, it is better to use the difference of position and velocity than relative motion because of physical sense
- Linear transformations between orbital elements and differences of position and velocity are accurate enough for its use in mission and analysis and control

$$\Delta EO = M^{-1}(EO)\Delta \vec{x}$$

$$\Delta \vec{x} = M(EO)\Delta EO$$

- Linear approximation of perturbations to predict temporal evolution of orbital elements differences could be problematic

$$\Delta EO = \frac{df(EO_0, t)}{dEO_0}\Delta EO_0$$